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PREDICTION OF TUMOUR MOTION USING INTERACTING MULTIPLE MODEL FILTER

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Abstract: Accurate prediction of tumour motion - over a prescribed time - is essential for enabling adaptive radiotherapy. The prediction time horizon is determined by measurement processing time, predictor algorithm processing time and the time-to-adapt radiation delivery. A trade off between the predictor algorithm complexity and the required prediction time horizon, therefore, has to be made. This paper proposes an interacting multiple model (IMM) filter and two Kalman filters to predict 0.2 s ahead respiratory tumour motions. The performance of the filters is evaluated using 333 traces of 4 minutes respiratory motions for 24 adult patients. The average RMSE of the IMM filter and the best Kalman filter with 5Hz measurements rate are 0.98 mm and 1.1 mm, which are improvements of 38% and 30% compared to use of measurements only.

Keywords: tumour motion prediction methods, Kalman filter, interacting multiple model filter, medical systems.

INTRODUCTION

Radiation therapy aims to precisely deliver a lethal dose to the tumour whilst minimizing radiation dose to surrounding healthy tissues. Recent developments in adaptive radiation therapy offer the possibility to dynamically change the shape of the radiation beams using multileaf collimators (MLC) [8] or to modify the patient position using a motorised patient support system (PSS). Existing radiotherapy treatment suites were not designed to have 'fast' response times. A typical image guided treatment delivery system exhibits time delays in the order of 0.2s [3]. To enable the radiotherapy equipment to adapt on-line to the tumour motion, accurate prediction methods to compensate the delay is therefore required [6].

Currently, there are limited methods for identification of organ/tumour and patient motion: internal fiducial markers inserted in the tumour, external markers positioned onto the patients or a combination of thereof [1]. A number of approaches have been designed to predict tumour motion. Sharp *et. al.* [6] compared Kalman filter, linear predictors and artificial neural networks to predict respiratory tumour motions. Neural networks were found to offer the best prediction with the Kalman filter being the worst in terms of root mean square errors (RMSE). The best RMSE was about 2mm for 0.2s ahead prediction using 30Hz sampled data. Vedam *et. al.* [7] investigated prediction performance of a sinusoidal predictor and an adaptive linear predictor and found that the adaptive predictor performs better. Average errors less than 2mm are achieved for prediction time less than 0.4s ahead with 10Hz measurements rate.

The aim of this paper is to propose a multiple model approach using the IMM filter algorithm for tumour motion prediction and to compare its performance to a Kalman filter for 0.2s prediction time horizon. The proposed Kalman filter is different from the one in [6] that it is based on linear stochastic models with the sampling period being the only parameter in the system matrices. The manuscript is organized as follows. Section 2 explains the proposed models, Kalman filter and IMM filter algorithms. The results of the prediction methods are discussed in section 3. Finally, conclusions are provided in section 4.

MODELS AND FILTER ALGORTIHMS

In order to use the Kalman filter and the IMM filter to predict tumour motions, models of the motions are needed. Several models have been proposed to mimic tumour motions, see [6, 9] and references therein. This paper proposes two stochastic linear models and a hybrid combination of them, which are suitable to use in both the Kalman filter and the IMM filter.

The first model is a constant velocity (CV) model

$$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix} + \begin{bmatrix} \Delta t \\ 1 \end{bmatrix} v(k-1) \quad (1)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + w(k), \quad (2)$$

where x_1 and x_2 denote the position and the velocity of the tumour, Δt the sampling period, y the measured tumour position, v and w the process and measurement noises that are assumed to be uncorrelated zero-mean Gaussian white noises with covariance matrices Q and R, respectively. The second model is a constant acceleration (CA) model the form

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \end{bmatrix} + \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \\ 1 \end{bmatrix} v(k-1) \quad (3)$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(k) + w(k), \quad (4)$$

which is an extension of (1)-(2) by including the acceleration denoted by x_3. It is important to note that the noise v is added to allow the changing of direction of the velocity and the acceleration such that the model is able to mimic both regular and irregular motions of the tumour. However, the CV and CA models are only valid for relatively small time steps, i.e. $0 < \Delta t < 0.5$s, because linear motions are assumed.

Since respiratory tumour trajectories may exhibit irregular motions as shown in [9], a single CV model or a single CA model may not be able to capture the

dynamics in the complete trajectory. For this reason, a hybrid combination of the two models is proposed. Hybrid systems are characterized by multiple models that describe various behaviour modes. In each mode there is a 'base state' and a 'modal state', which indicates in what mode the system is at a certain time. If (1)-(2) and (3)-(4) are rewritten as

$$\vec{x}_j(k) = F_j\vec{x}_j(k-1) + G_j v_j(k-1),\ j \in \{1,2\} \quad (5)$$

$$y_j(k) = H_j\vec{x}_j(k) + w_j(k), j \in \{1,2\}, \quad (6)$$

the proposed hybrid system is given by

$$x(k) = \sum_{j=1}^{2} \mu_j(k-1)(F_j\vec{x}_j(k-1) + G_j v_j(k-1)) \quad (7)$$

$$y(k) = \sum_{j=1}^{2} \mu_j(k)(H_j\vec{x}_j(k) + w_j(k)) \quad (8)$$

$$\mu(k) = \Pi^T \mu(k-1), \quad (9)$$

where μ is the modal state with its element μ_j the probability being in mode j and Π is the Markovian transition matrix with its element π_{ij} the probability of the transition from being in mode i at step *k-1* to being in mode j at step k. Both μ_j and π_{ij} take values between 0 and 1 such that the hybrid system allows soft switching between the two local models.

Kalman filter

The Kalman filter is an optimal state estimator of linear systems that minimizes the mean of the squared error of the estimation [4]. The recursive feature of Kalman filter makes it suitable for online prediction. It is widely used for target tracking and autonomous navigation. The Kalman filter algorithm consists of prediction and update steps, for details see the filtering step of the IMM algorithm and [4, 6].

Interacting multiple model (IMM) filter

The IMM filter [2] is a suboptimal state estimator for hybrid systems, for example given by (7)-(9), which has been successfully applied for tracking manoeuvring targets in airborne navigations. The IMM filter uses a Kalman filter as the base state estimator for each of local models and utilizes the normalized likelihood of those Kalman filters to estimate the modal state. The IMM algorithm consists of three steps in each iteration. For s local models, i.e. $M_s = \{1,2,...,s\}$, the algorithm reads as follows:

a) Interaction $(\forall i, j \in M_s)$

Mode probability prediction

$$\mu_j(k|k-1) = \sum_i \pi_{ij}\mu_i(k-1)$$

Mixing probability $\mu_{i|j}(k|k-1) = \dfrac{\pi_{ij}\mu_i(k-1)}{\mu_j(k|k-1)}$

Initialization of local filters

$$\hat{x}_{0j}(k|k-1) = \sum_i \hat{x}_i(k-1)\mu_{i|j}(k|k-1)$$

$$P_{0j}(k|k-1) = \sum_i \{P_i(k-1) + [\hat{x}_i(k-1) - \hat{x}_{0j}(k|k-1)]$$

$$[\hat{x}_i(k-1) - \hat{x}_{0j}(k|k-1)]^T\}\mu_{i|j}(k|k-1)$$

b) Filtering [Kalman filter] $(\forall i, j \in M_s)$

Prediction $\hat{x}_j(k|k-1) = F_j\hat{x}_{0j}(k|k-1)$

$$P_j(k|k-1) = F_j P_{0j}(k-1)F_j^T + G_j Q_j G_j^T$$

Predicted target position for output:

$$\hat{y}(k|k-1) = \sum_j \mu_j(k-1)H_j\hat{x}_j(k|k-1)$$

Residual $r_j(k) = y(k) - \hat{y}_j(k|k-1)$

$$S_j(k) = H_j P_j(k|k-1)H_j^T + R_j$$

Kalman gain $K_j(k) = P_j(k|k-1)H_j^T S_j^{-1}$

Update $\hat{x}_j(k) = \hat{x}_j(k|k-1) + K_j(k)r_j(k)$

$$P_j(k) = P_j(k|k-1) - K_j(k)S_j(k)K_j(k)^T$$

Likelihood $\Lambda_j = \dfrac{1}{\sqrt{2\pi S_j(k)}}\exp\left(-\dfrac{r_j(k)^2}{2S_j(k)}\right)$

Mode probability $\mu_j(k) = \dfrac{\Lambda_j(k)\mu_j(k|k-1)}{\sum_i \Lambda_i(k)\mu_i(k|k-1)}$

c) Combination $(\forall i, j \in M_s)$

$$\hat{x}(k) = \sum_j \hat{x}_j(k)\mu_j(k)$$

$$P(k) = \sum_j \{P_j(k) + [\hat{x}_j(k) - \hat{x}(k)][\hat{x}_j(k) - \hat{x}(k)]^T\}\mu_j(k).$$

The iteration is initialized with $\hat{x}_j(0) = \bar{x}_j(0)$, $P_j(0) = \bar{P}_j(0)$, $\mu_j(0) = \bar{\mu}_j(0)$ for all $j \in M_s$. In the algorithm, $\hat{x}_j(k)$ and $P_j(k)$ are the estimated state of model j and its covariance, $\mu_j(k|k-1)$ is the predicted probability of mode j at step k given measurement up to $y(k-1)$, π_{ij} is the element of the Markovian transition matrix Π governing the transition of the mode probability, and $\mu_{i|j}(k|k-1)$ is the mixing probability, which is the weight for the estimate of filter i at step *k-1* for the initialization of filter j at step k.

RESULTS AND DISCUSSION

The prediction performance of Kalman filters with CV and CA models and IMM filter is tested using the data acquired from a breathing training database collated at Virginia Commonwealth University, USA. 24 adult patients suffering from lung cancer were observed over a period of a year, a collection being made of 333 4-minute breathing traces of respiratory motion. A marker block resting on the chest of each patient between umbilicus and xyphoid allowed tracking of respiratory movement in the anterior-posterior (AP) direction using

a real-time position management system developed by Varian Medical Systems. This method gives a reasonable analogue of AP tumour movement without the requirement for invasive procedures to provide imaging. Although Berbeco *et. al.* [1] shows that correlation between external marker position and lung tumour position can be subject to error, the present study is limited to predicting marker position only. In the original data the sampling rate is 30Hz but in this paper the data is down-sampled to 5Hz to accommodate for the 0.2s prediction, i.e. $\Delta t = 0.2$s.

Firstly, the prediction of the filters to track different types of tumour trajectories is compared and later applied to the whole data set. Design parameters of the Kalman filter with the CV model (Kalman CV) and Kalman filter with the CA model (Kalman CA) are set to $Q_1 = 90$ cm^{2}s^{-2}, $R_1 = 10^{-4}$cm^2, $Q_2 = 0.1$ cm^{2}s^{-4} and $R_2 = 10^{-4}$cm^2, respectively. The same Kalman CV and Kalman CA filters are used as local filters in the IMM algorithm and the Markovian transition matrix is set to $[\,]$=[0.9 0.1; 0.5 0.5] meaning that the trajectories are assumed to be more frequently in the CV mode. The Kalman CV is initialized with $x_1(0) = y(1)$, $x_2(0) = (y(1) - y(2))/T$, and $P(0) = I_2$, and the Kalman CA is initialized with the extra state $x_3(0) = ((y(3)-y(2))/T - x_2(0))/T$ and $P(0) = I_2$, where I_2 and I_3 are 2-dimensional and 3-dimensional identity matrices, respectively.

Table 1 shows that the Kalman CV gives better prediction than the Kalman CA for the trajectory shown in Fig. 1. However, Table 2 indicates that the Kalman CA provides better prediction for the trajectory depicted in Fig. 2. This illustrates that none of them is able to perform equally well on all possible tumour motion in the data set. Combining both filters using the IMM algorithm gives better prediction with the exception of Trajectory No.1, see Table 3. The degradation of the IMM filter performance is caused by the poor performance of the Kalman CA. Nevertheless, Table 3 also shows that the IMM filter and the Kalman CV are still able to provide better prediction than using measurements only (no prediction) even for the tumour trajectory with an abrupt change of 3cm displacement, as depicted in Fig. 3.

Table 2 shows that all three filters have larger maximum absolute error than the no prediction whilst all of them have much smaller RMSE. This phenomenon occurs at the time instant 76s where accidentally the delayed data is closer to the actual data than the predictions of the filters, see Fig. 4.

For all of the data set, the propose filters are able to improve the prediction performance compare to no prediction as shown in Table 4. The IMM filter give the best prediction that reduces the average RMSE by 38%, while the Kalman CV and the Kalman CA give 30% and 27% reduction, respectively. Furthermore, Table 5 indicates that the prediction performance of the IMM filter and the Kalman CV can be improved if the measurements rate is increased to 10Hz.

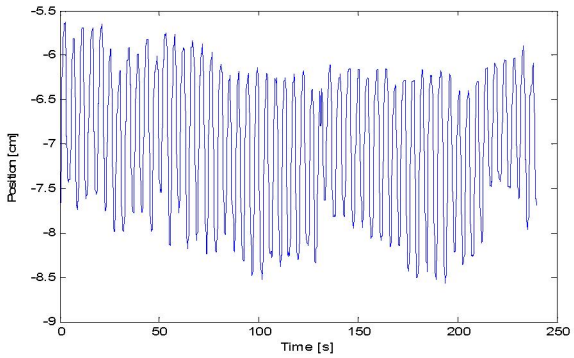

Fig. 1: Trajectory No.5 of the Virginia data set showing quasi-periodic motion

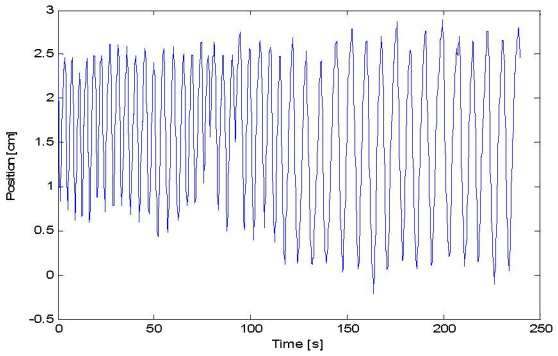

Fig. 2: Trajectory No. 58 of the Virginia data set showing changing of amplitude and frequency.

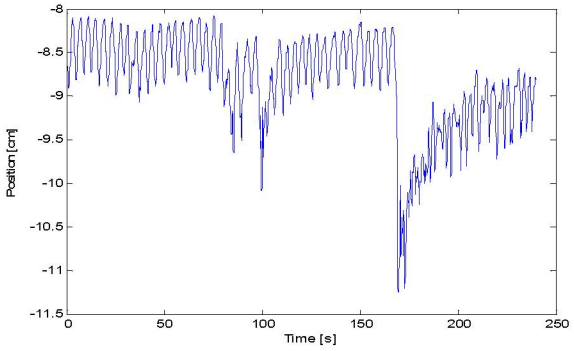

Fig. 3: Trajectory No. 1 of the Virginia data set showing abrupt transition and irregular motion.

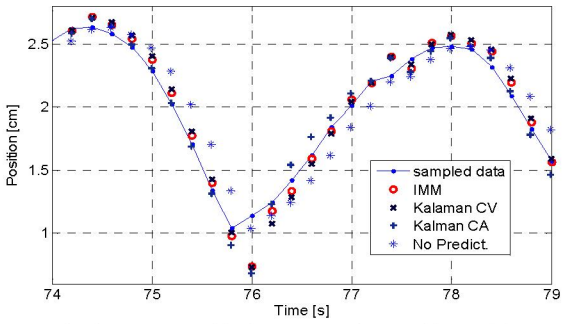

Fig. 4: The largest prediction error showed in Table 2 occurs at time t = 76s of Trajectory No. 5.

Methods	RMSE		Max |*error*|	
	[mm]	gain [%]	[mm]	gain [%]
No Predict.	2.16	reference	5.80	reference
Kalman CV	1.27	41.40	4.16	28.34
Kalman CA	1.30	40.10	5.41	6.71
IMM	1.00	49.17	4.00	31.41

Table 1: Filters performance to predict Trajectory No. 5 0.2s ahead with 5Hz measurements rate.

Methods	RMSE		Max |*error*|	
	[mm]	gain [%]	[mm]	gain [%]
No Predict.	1.65	reference	3.77	reference
Kalman CV	0.86	48.10	4.08	-8.25
Kalman CA	0.82	50.31	4.51	-19.65
IMM	0.76	53.94	4.01	-6.41

Table 2: Filters performance to predict Trajectory No. 58 0.2s ahead with 5Hz measurements rate.

Methods	RMSE		Max |*error*|	
	[mm]	gain [%]	[mm]	gain [%]
No Predict.	1.16	reference	11.87	reference
Kalman CV	1.12	2.76	9.76	17.80
Kalman CA	1.38	-19.13	13.38	-12.78
IMM	1.14	1.36	10.39	12.43

Table 3: Filters performance to predict Trajectory No. 1 0.2s ahead with 5Hz measurements rate.

Methods	RMSE [mm]		
	Min	Max	Average
No Predict.	0.69	3.91	1.58
Kalman CV	0.29	3.31	1.10
Kalman CA	0.28	3.84	1.16
IMM	0.28	3.08	0.98

Table 4: Filters performance for 0.2s ahead prediction with 5Hz measurements rate for the whole data set.

Methods	RMSE [mm]		
	Min	Max	Average
No Predict.	0.68	3.901	1.58
Kalman CV	0.28	2.94	0.95
Kalman CA	0.29	4.01	1.16
IMM	0.30	2.81	0.92

Table 5: Filters performance for 0.2s ahead prediction with 10Hz measurements rate for the whole data set.

CONCLUSIONS

Two discrete-time linear stochastic models, i.e. CV and CA models, and a hybrid combination of the two are proposed to mimic respiratory tumour motions. These models have been found to be suitable for predicting tumour motions 0.2s ahead using both Kalman filter and IMM filter. The performance of the Kalman filter with CV and CA models and the IMM filter has been tested using a clinical data set containing 333 traces of 4 minutes respiratory tumour motions. The IMM filter gave the best prediction with average RMSE 0.98mm, which is a reduction of 38% compared to using measurements only, for 5Hz measurement rate. If the measurement rate is increased to 10Hz, the average RMSE is reduced to 0.92mm.

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